Course of Study

Calculus

Warren County Career Center

3525 North State Route 48
Lebanon, Ohio 45036

Adopted 09/18/2008

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Acknowledgements

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We would like to take this opportunity to express our gratitude to the following people for their guidance and support in the preparation of this course of study:

Warren County Career Center Administrative Team
Warren County Educational Service Center
Chuck Goodyear

Warren County Career Center
Resolution Of School Board Approval

WHEREAS, representatives of the Math Department of the Warren County Career Center have reviewed the Calculus Course of Study; and

WHEREAS, this Course of Study is based upon Integrating Technical and Academic Competencies adopted by the State of Ohio for the Calculus program; and

WHEREAS, the Math faculty and the Career-Technical Advisory Committee have reviewed and added competencies as needed to address local labor market needs and trends in the industry;

NOW, THEREFORE, BE IT RESOLVED, in accordance with the Superintendent's recommendation, that the Warren County Career Center adopt the Calculus Course of Study.

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District Superintendent Date

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President, Board of Education Date

Statement of Recommendation

The Calculus at Warren County Career Center has reviewed this course of study and recommends it for use as the foundation for instruction in the Calculus class.

The developers of this course of study have considered local labor market needs and the school's ability to offer specialized programs. The competencies have been reviewed and accepted as being congruent with our school's vision, mission, and strategic goals. When appropriate, additional competencies related to the program area have been incorporated into this course of study.

Achievement of technical competencies, utilizing proper attitudes, and demonstrating appropriate values are critical for successful employment and for furthering educational opportunities within a student's chosen field. We believe that this course of study adequately and correctly focuses upon student development.

This course of study is recommended on: September 18, 2008

Warren County Career Center Vision Statement

WCCC is the valued partner of choice within the educational and economic systems of our communities, by providing quality academic and career technical education.

We pave the way for a future of opportunities unique to each of our learners.

Warren County Career Center Mission Statement

To prepare youths and adults to make informed career choices and to successfully enter, compete, and advance in a changing work world.

Warren County Career Values

- Treating each other with respect, dignity, trust and mutual value
- Communicating openly and honestly
- Taking ownership of personal actions and being held accountable for results
- Upholding and demonstrating high ethical, educational and fiscal standards
- Exhibiting high levels of professionalism
- Providing high quality instruction and highly qualified staff to ensure success for all learners
- Making quality customer service a high priority
- Promoting partnerships and a team environment
- Celebrating team and individual achievements
- Using data to drive planning, decision making and actions
- Embracing educational opportunities for change and diversity

Course Design

Courses are designed to reflect career-focused education, which combines high-level academics with real-life technical skills. The intent is to maximize a student's present and future academic and career success.

Career-focused education enhances the integration of academic and technical skills, designs programs that prepare students with transferable skills and promotes each student's career opportunities.

Course Philosophy

We believe that Calculus is primarily concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications. The courses emphasize a multi-representational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations also are important.

Broad concepts and widely applicable methods are emphasized. The focus of the course is neither manipulation nor memorization of an extensive taxonomy of functions, curves, theorems, or problem types. Thus, although facility with manipulation and computational competence are important outcomes, they are not the core of these courses.

Technology should be used regularly by students and teachers to reinforce the relationships among the multiple representations of functions, to confirm written work, to implement experimentation, and to assist in interpreting results.

Through the use of the unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes are developed using all the functions listed in the prerequisites.

Course Goals

The course goals for Calculus are to:

- Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation and should be able to use derivatives to solve a variety of problems.
- Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change and should be able to use integrals to solve a variety of problems.
- Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- Students should be able to communicate mathematics and explain solutions to problems both verbally and in written sentences.
- Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.
- Students should be able to use technology to help solve problems, experiment, interpret results, and support conclusions.
- Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- Students should develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Course Description

Calculus consists of a full high school academic year of work and is comparable to calculus courses in colleges and universities. It is expected that students who take a course in calculus will seek college credit, college placement, or both, from institutions of higher learning.

Calculus can be offered as a course by any school that can organize a curriculum for students with mathematical ability. Calculus is designed to be taught over a full high school academic year. It is possible to spend some time on elementary functions and still cover the Calculus curriculum within a year. However, if students are to be adequately prepared for the Calculus Exam, most of the year must be devoted to the topics in differential and integral calculus.

Academic and Technical Integration

Expectations of curriculum must be aligned with what is written, taught, assessed, and reported. Student expectations focus on active, project-centered learning—an approach to learning that emphasizes a connection between ideas in a discipline and the outside world. Educational programming and course content will clearly connect career and post-secondary opportunities. At the Warren County Career Center, the main goal is to design courses and projects that use strategies for authentic instruction. These characteristics of instruction focus on deep understanding, established opportunities for concept connections, provide anticipatory and abstract thinking, and emphasize genuine application.

The academic courses at the WCCC follow the state model curricula. They are designed to meet both associate school and state requirements. These standards respond to the need to improve student achievement, quality of curriculum and instruction, and strengthen school and community relationships.

Technology

The Warren County Career Center board and staff believe that technology skills are essential for all students to achieve in the 21st century. It is the goal of this district to infuse technology into all facets of education:

- Instruction
- Assessment
- Administration
- Career planning
- Course design
- Professional development

Strategies to incorporate technology into all facets of education are a priority of the district and there is commitment to a continual process to provide updated hardware, software, and professional development for staff members for the purpose of providing a high quality education, with the integration of technology, for all students.

Students Served

The population served by this program is seniors.

Scope and Sequence

Topic Outline for Calculus

This topic outline is intended to indicate the scope of the course, but it is not necessarily the order in which the topics need to be taught. Teachers may find that topics are best taught in different orders. Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and Limits

Analysis of graphs With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

- An intuitive understanding of the limiting process
- Calculating limits using algebra
- Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior
- Describing asymptotic behavior in terms of limits involving infinity
- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)

Continuity as a property of functions

- An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

II. Derivatives

Concept of the derivative

- Derivative presented graphically, numerically, and analytically
- Derivative interpreted as an instantaneous rate of change
- Derivative defined as the limit of the difference quotient
- Relationship between differentiability and continuity

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation
- Instantaneous rate of change as the limit of average rate of change
- Approximate rate of change from graphs and tables of values

Derivative as a function

- Corresponding characteristics of graphs of f and f'

- Relationship between the increasing and decreasing behavior of f and the sign of f'
- The Mean Value Theorem and its geometric interpretation
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of the graphs of f, f', and f''
- Relationship between the concavity of f and the sign of f''
- Points of inflection as places where concavity changes

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity
- Optimization, both absolute (global) and relative (local) extrema
- Modeling rates of change, including related rates problems
- Use of implicit differentiation to find the derivative of an inverse function
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- Derivative rules for sums, products, and quotients of functions
- Chain rule and implicit differentiation

III. Integrals

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

- Basic properties of definite integrals (examples include additivity and linearity)

Applications of integrals Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen , the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change.

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined

Techniques of anti-differentiation

- Anti-derivatives following directly from derivatives of basic functions
- Anti-derivatives by substitution of variables (including change of limits for definite integrals)

Applications of anti-differentiation

- Finding specific anti-derivatives using initial conditions, including applications to motion along a line
- Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth)

Numerical approximations to definite integrals Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

Technology Standards

Standard 1: Nature of Technology

Students develop an understanding of technology, its characteristics, scope, core concepts* and relationships between technologies and other fields.

Benchmark A: Synthesize information, evaluate and make decisions about technologies.

Benchmark B: Apply technological knowledge in decision-making.

Benchmark C: Examine the synergy between and among technologies and other fields of study when solving technological problems.

Standard 2: Technology and Society Interaction

Students recognize interactions among society, the environment and technology, and understand technology's relationship with history. Consideration of these concepts forms a foundation for engaging in responsible and ethical use of technology.

Benchmark A: Interpret and practice responsible citizenship relative to technology.

Benchmark B: Demonstrate the relationship among people, technology and the environment.

Benchmark C: Interpret and evaluate the influence of technology throughout history, and predict its impact on the future.

Benchmark D: Analyze ethical and legal technology issues and formulate solutions and strategies that foster responsible technology usage.

Benchmark E: Forecast the impact of technological products and systems.

Standard 3: Technology for Productivity Applications

Students learn the operations of technology through the usage of technology and productivity tools.

Benchmark A: Integrate conceptual knowledge of technology systems in determining practical applications for learning and technical problem-solving.

Benchmark B: Identify, select and apply appropriate technology tools and resources to produce creative works and to construct technology-enhanced models.

Standard 4: Technology and Communication Applications

Students use an array of technologies and apply design concepts to communicate with multiple audiences, acquire and disseminate information and enhance learning.

Benchmark A: Apply appropriate communication design principles in published and presented projects.

Benchmark B: Create, publish and present information, utilizing formats appropriate to the content and audience.

Benchmark C: Identify communication needs, select appropriate communication tools and design collaborative interactive projects and activities to communicate with others, incorporating emerging technologies.

Standard 5: Technology and Information Literacy

Students engage in information literacy strategies, use the Internet, technology tools and resources, and apply information-management skills to answer questions and expand knowledge.

Benchmark A: Determine and apply an evaluative process to all information sources chosen for a project.

Benchmark B: Apply a research process model to conduct research and meet information needs.

Benchmark C: Formulate advanced search strategies, demonstrating an understanding of the strengths and limitations of the Internet, and evaluate the quality and appropriate use of Internet resources.

Benchmark D: Evaluate choices of electronic resources and determine their strengths and limitations.

Standard 6: Design

Students apply a number of problem-solving strategies demonstrating the nature of design, the role of engineering and the role of assessment.

Benchmark A: Identify and produce a product or system using a design process, evaluate the final solution and communicate the findings.

Benchmark B: Recognize the role of teamwork in engineering design and of prototyping in the design process.

Benchmark C: Understand and apply research, development and experimentation to problem-solving.

Standard 7: Designed World

Students understand how the physical, informational and bio-related technological systems of the designed world are brought about by the design process. Critical to this will be students' understanding of their role in the designed world: its processes, products, standards, services, history, future, issues and career connections.

Benchmark A: Classify, demonstrate, examine, and appraise energy and power technologies.

Benchmark B: Classify, demonstrate, examine and appraise transportation technologies.

Benchmark C: Classify, demonstrate, examine and appraise manufacturing technologies.

Benchmark D: Classify, demonstrate, examine and appraise construction technologies.

Benchmark E: Classify, demonstrate, examine and appraise information and communication technologies

Benchmark F: Classify, demonstrate, examine and appraise medical technologies.

Benchmark G: Classify, demonstrate, examine and appraise agricultural and related biotechnologies.

Performance Measures/Student Assessment/Instructional Strategies

Assessments/Evaluations

- Observations
- Demonstrations
- Portfolios
- Standardized Tests
- Class Assignment
- Quizzes/Tests/Exams
- Web Exam/Certification

Instructional Strategies

- Teacher-Directed & Student-Centered Activities
- Case Study Problem Solving
- Cooperative Learning
- Project-Based Learning
- Career-Based Learning (Internships/Shadowing/Placement)
- Community-Based Learning (CTSOs and Other)
- Exploratory Learning
- Independent Research
- Team Teaching

Content Specific Strategies